Test Paper : **II**	Test Booklet Serial No. : ____________
Test Subject : **MATHEMATICAL SCIENCES**	OMR Sheet No. : ____________
Test Subject Code : **A-15-02**	Hall Ticket No. ☐☐☐☐☐☐☐☐☐ (Figures as per admission card)

Name & Signature of Invigilator

Name : ____________ Signature : ____________

Paper : II
Subject : MATHEMATICAL SCIENCES

Time : 1 Hour 15 Minutes | Maximum Marks : 100

Number of Pages in this Booklet : **8** | Number of Questions in this Booklet : **50**

Instructions for the Candidates

1. Write your Hall Ticket Number in the space provided on the top of this page.
2. This paper consists of fifty multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
   (iii) After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Sheet given to you**. If you mark at any place other than in the circle in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test question booklet and OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

అభ్యర్థులకు సూచనలు

1. ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్ టికెట్ నంబరు రాయండి.
2. ఈ ప్రశ్న పత్రము యాభై బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
3. పరీక్ష ప్రారంభమున ఈ ప్రశ్నాపత్రము మీకు ఇవ్వబడుతుంది. మొదటి ఐదు నిమిషములలో **ఈ ప్రశ్నాపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా సరిచూసుకోండి.**
   (i) ఈ ప్రశ్న పత్రమును చూడడానికి కవర్‌పేజి అంచున ఉన్న కాగితపు సీలును చించండి. స్టిక్కర్ సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నాపత్రమును మీరు అంగీకరించవద్దు.
   (ii) **కవరు పేజి పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్నపత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏవైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇచ్చివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.**
   (iii) పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నాపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నాపత్రము పై నిర్దిష్టస్థలంలో రాయవలెను.
4. ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయ ప్రతిస్పందనలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతిప్రశ్నకు సరైన ప్రతిస్పందనను ఎన్నుకొని కింద తెలిపిన విధంగా OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన ప్రతిస్పందనను సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్‌తో కింద తెలిపిన విధంగా పూరించాలి.
   ఉదాహరణ : (A) (B) ● (D)
   (C) సరైన ప్రతిస్పందన అయితే
5. ప్రశ్నలకు ప్రతిస్పందనలను ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమాధాన పత్రంపై వేరొక చోట గుర్తిస్తే మీ ప్రతిస్పందన మూల్యాంకనం చేయబడదు.
6. ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
7. చిత్తుపనిని ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీస్థలములో చేయాలి.
8. OMR పత్రము పై నిర్ణీత స్థలంలో సూచించవలసిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అనర్హతకు మీరే బాధ్యులవుతారు.
9. పరీక్ష పూర్తయిన తర్వాత మీ ప్రశ్నపత్రాన్ని మరియు OMR పత్రాన్ని తప్పనిసరిగా పరీక్షపర్యవేక్షకుడికి ఇవ్వాలి. వాటిని పరీక్ష గది బయటకు తీసుకువెళ్లకూడదు.
10. **నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.**
11. **లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్‌లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్షగదిలో ఉపయోగించడం నిషేధం.**
12. **తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.**

MATHEMATICAL SCIENCES

Paper – II

1. Define $f_n = \frac{x^2}{(1+x^2)^n}$, x is real, n = 0, 1, 2, … , then for $x \neq 0$, $\sum_{n=0}^{\infty} f_n(x) =$

(A) 0 (B) 1

(C) 1 + x (D) $1 + x^2$

2. The function f(x) = sin(1/x), x > 0 is

(A) Uniformly continuous on R^+

(B) Uniformly continuous on R

(C) Continuous on R^+

(D) Not continuous on R^+

3. $\int_0^1 x\,(1-x^2)^n\,dx =$

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{1}{2(n+1)}$ (D) $\frac{1}{(2n+1)}$

4. If p > 0 and $\alpha \in R$, then $\lim_{n\to\infty} \frac{n^\alpha}{(1+p)^n} =$

(A) 0 (B) 1

(C) ∞ (D) not defined

5. If T: $R^3 \to R^2$ is a linear transformation defined by T (x, y, z) = (x + 2y – z, y + z, x + y – 2z), then the rank of T is equal to

(A) 0 (B) 1 (C) 2 (D) 3

6. Let A and B be n×n matrices with real numbers as elements. Then which one of the following statements is NOT true ?

(A) Rank of A = Rank of A′

(B) Rank of (AB) = Rank of (BA)

(C) Rank of (A + B) ≤ Rank of A + Rank of B

(D) If C is obtained after a finite number of elementary operations on A, then Rank of A = Rank of C

7. The largest eigen value of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix} \text{ is}$$

(A) 2 (B) 3 (C) 4 (D) 6

8. Let T: $R^3 \to R$ be a linear transformation defined by T (1,1, 1) = 3, T (0, 1,– 2) = 1 and T (0, 0, 1) = – 2

Then T (x, y, z) =

(A) 8x + 3y – 2z (B) 3x – 8y – 2z

(C) 8x – 3y – 2z (D) 2x – 3y + 8z

9. If f (z) = u (x, y) + iv (x, y) and g (z) = u (x, y) – iv (x, y) are analytic functions defined on the same domain and f (1 + i) = 2 + 3i, then g (4 + 3i) =

(A) 4 + 3i (B) 4 – 3i

(C) 2 + 3i (D) 2 – 3i

10. If an entire function f (z) is bounded in absolute value for all z, then f (z) =

(A) Constant (B) e^z

(C) 1/z (D) z

11. If f (z) = sinz + e^{3z}, then $\frac{\partial f(z)}{\partial \bar{z}} =$

(A) 0 (B) $3e^{3\bar{z}}$

(C) $\cos\bar{z}$ (D) $\cos\bar{z} + 3e^{3\bar{z}}$

12. The residue of f (z) = $\frac{z^2+3z+2}{z^2(z-1)}$ at z = 0 is

(A) 0 (B) – 1 (C) 2 (D) – 5

13. If G is a finite abelian group, then G is isomorphic to the direct product of its
(A) normal subgroups
(B) abelian subgroups
(C) sylow subgroups
(D) cyclic subgroups

14. Which one of the following is NOT true?
(A) There are $\phi(n)$ premature n-th roots of unity where $\phi(n)$ is the Euler ϕ- function
(B) If ω is a primitive n-th root of unity, then $F_0(\omega)$ is the splitting field of $x^n - 1$ over F_0
(C) If $\omega_1, \omega_2, ..., \omega_{\phi(n)}$ are the $\phi(n)$ primitive n-th roots of unity, then any automorphism of $F_0(\omega_1)$ takes ω_1 into some ω_i
(D) $[F_0(\omega_1) : F_0] > \phi(n)$

15. If R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/M is
(A) a quotient ring (B) a field
(C) a division ring (D) an ideal

16. The solution of $x^2 \oplus 1 = 0$ in the field $(Z_5, \oplus, \bullet)$ is
(A) 0, 1 (B) 1, 2 (C) 2, 3 (D) 3, 4

17. The number of units in the domain of Gaussian integers is
(A) 1 (B) 2 (C) 3 (D) 4

18. Consider the linear space X = C [0, 1] with norm $\|f\| = \sup\{|f(t)| : 0 \le t \le 1\}$. Let $F = \{f \,\varepsilon\, X : f(½) = 0\}$ and $G = \{g \,\varepsilon\, X : g(½) \neq 0\}$. Then
(A) F is not closed and G is open
(B) F is closed but G is not open
(C) F is not closed and G is not open
(D) F is closed and G is open

19. An integrating factor for the differential equation $ydx + (x^2y - x)\,dy = 0$ is
(A) x (B) 1/x (C) $1/x^2$ (D) e^x

20. For the differential equation $y'' - x^2y' - xy = 0$ an equivalent system of first order equations is
(A) $dy/dx = y+z$; $dz/dx = x^2z + y$
(B) $dy/dx = z$; $dz/dx = xy + x^2z$
(C) $dy/dx = xz$; $dz/dx = y + xz$
(D) $dy/dx = x + z$; $dz/dx = y + x^2z$

21. The eigen functions of the Sturm – Liouville problem $x'' + \alpha x = 0$, $x(0) = 0$ and $x(1) = 0$, for $0 \le t \le 1$, n = 0, 1, 2, … are
(A) $x_n(t) = \sin(2n+1)\pi t$
(B) $x_n(t) = \cos(2n+1)\pi t/2$
(C) $x_n(t) = \sin(2n+3)\pi t/2$
(D) $x_n(t) = \cos(2n+3)\pi t/2$

22. The stationary function of $\int_0^4 (xy' - y'^2)\,dx$ subject to y(0) = 0 and y(4) = 3 is
(A) $y = ½ (x^2 - 1)$ (B) $y = ½ (x^2 + 1)$
(C) $y = ¼ (2x^2 + x)$ (D) $y = ¼ (x^2 - x)$

23. The general form of Hamiltonian equations of motion for a conservative holonomic dynamical system for k = 1, 2, 3, …, n are

(A) $\dot{q}_k = \frac{\partial H}{\partial t_k},\ \dot{p}_k = -\frac{\partial H}{\partial q_k}$

(B) $\dot{q}_k = -\frac{\partial H}{\partial t_k},\ \dot{p}_k = -\frac{\partial H}{\partial q_k}$

(C) $q_k = \frac{\partial H}{\partial t_k},\ p_k = -\frac{\partial H}{\partial t_k}$

(D) $q_k = \frac{\partial H}{\partial t_k},\ p_k = -\frac{\partial H}{\partial p_k}$

24. If the kinetic energy of a body of mass m moving in one-dimension is ½ $m\dot{x}^2$ and its potential energy is ½ kx^2 where k is a constant then its Lagrange's equations of motion is

(A) $k\ddot{x} + mx = 0$ (B) $m\ddot{x} + kx = 0$

(C) $m\dot{x} + kx = 0$ (D) $k\ddot{x} + m\dot{x} = 0$

25. The resolvent kernel $R(x, t; \lambda)$ for the $K(x, t) = x^2t^2$, a = –1, b = 1 is

(A) $\frac{3x^2t^2}{1-2\lambda}, |\lambda| < 1/2$ (B) $\frac{4x^2t^2}{3-2\lambda}, |\lambda| < 3/2$

(C) $\frac{5x^2t^2}{5-2\lambda}, |\lambda| < 5/2$ (D) $\frac{5x^2t^2}{5+2\lambda}, |\lambda| < 2/5$

26. The solution of the partial differential equation (y – z)p + (z – x)q = x – y is

(A) $f(x + y + z) = x^2 + y^2 + z^2$

(B) $f(x + y + z) = xy + yz + zx$

(C) $f(x^2 + y^2 + z^2) = xyz$

(D) $f(x^2 + y^2 + z^2) = x + y + z$

27. The solution of the partial differential equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, $u(x,0) = 4e^{-x}$ is u =

(A) $2e^{5x + 3y} + 2e^{-3x + 2y}$

(B) $3e^{-5x + 3y} + e^{3x - 2y}$

(C) $4e^{-(2x - 3y)/2}$

(D) $4e^{-(3x - 2y)/2}$

28. The Newton-Raphson method of finding a root of a polynomial or transcendental equation has the rate of convergence

(A) 3 (B) 2

(C) 1.67 (D) 1.4

29. An approximate solution for the system of linear equations
10x + 2y + z = 9, 2x + 20y – 2z = – 44 ;
– 2x + 3y + 10z = 22 using Gauss-Seidal method is

(A) x = 1.094, y = – 1.908, z = 2.946

(B) x = 1.082, y = – 1.914, z = 2.965

(C) x = 1.108, y= – 1.932, z = 2.981

(D) x = 1.013, y = – 1.996, z = 3.001

30. The value of the integral $I = \int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3 rule, is

(A) 1.2886 (B) 1.3268

(C) 1.3662 (D) 1.3874

31. For any discrete distribution

(A) S.D ≥ M.D from mean

(B) S.D = M.D

(C) S.D < M.D from mean

(D) No relation between S.D and M.D

32. For a random variable x moments of all order exit and μ_j denote j[th] central moment, then

(A) $\mu_{2j+1}^{2} > \mu_{2j}\,\mu_{2j+2}$

(B) $\mu_{2j+1}^{2} < \mu_{2j}\,\mu_{2j+2}$

(C) $\mu_{2j+1} = \mu_{2j}\,\mu_{2j+2}$

(D) either B or C is correct

33. The probability distribution of a random variable X is f(x) = k sin(πx/5), $0 \leq x \leq 5$, then the ratio between median of X and k is

(A) 25:π (B) 5:2

(C) 1:2 (D) 10:π

34. If ϕ(t) denotes the characteristic function of a random variable, then

i) $\phi(0) = 1$,

ii) $\phi(t) \leq 1$,

iii) ϕ(t) is continuous every where

(A) only i) is correct but not ii) and iii)

(B) only ii) is correct but not iii)

(C) all i), ii) and iii) are correct

(D) none of i), ii) and iii) are correct

35. If you wish to estimate the proportion of engineers and scientists who have studied statistics and you wish your estimate to be correct within 2% with probability 0.95 or more, how large sample would you take if you have no idea what the true proportion is ?

(A) 1200 (B) 800

(C) any number (D) 95

36. The Poisson process satisfies the following conditions

(A) regularity

(B) homogeneity

(C) independent increments

(D) all the above three (A), (B) and (C)

37. For a Binomial distribution with p = 0.5 has maximum probability when

(A) X = n/2 if n is even

(B) x = ½ (n – 1) and x = ½ (n + 1) if n is odd

(C) x = n

(D) both A) and B) are correct

38. The probability mass function of a Poisson variate with parameter λ truncated at 0 is

(A) $\frac{e^{-\lambda}\lambda^{x}}{x!}$ (B) $\frac{e^{-\lambda}\lambda^{x}}{x!}\left(1-e^{-\lambda}\right)$

(C) $\frac{e^{-\lambda}\lambda^{x}}{x!\left(1-e^{-\lambda}\right)}$ (D) $1-e^{-\lambda}$

39. Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from an exponential distribution with parameter λ, then the distribution function of $Y = \min(x_1, x_2, \ldots, x_n)$ is

(A) $n(n-1)\left(1-e^{-n\lambda y}\right)$

(B) $\left(1-e^{-n\lambda y}\right)$

(C) $\frac{1}{2}n(n-1)\left(1-e^{-n\lambda y}\right)$

(D) $\left(1-e^{-\lambda y}\right)$

40. The minimum variance bound estimator for θ in a Cauchy population with parameter θ is
(A) sample mean
(B) sample median
(C) reciprocal of sample median
(D) does not exist

41. In a Poisson distribution with unit mean the mean deviation about mean is
(A) 2/e (B) e/2 (C) e (D) 1

42. With normal notations the variance of M.L.E. is
(A) $I(\theta)$ (B) $1/I(\theta)$
(C) $\sqrt{I(\theta)}$ (D) $-1/I(\theta)$

43. Neyman-Pearson lemma provides
(A) An unbiased test
(B) a most powerful test
(C) an admissible test
(D) sufficient test

44. With the usual notation the sum of squares due to any factorial effect is
(A) $\frac{()^2}{\sum \gamma_i c_i^2}$ (B) $\frac{()^2}{\sum \gamma_i^2 c_i}$
(C) $\frac{()^2}{\sum \gamma_i^2}$ (D) $\frac{()^2}{\sum \gamma_i c_i}$

45. The information on auxiliary variate is used in the following sampling scheme
(A) double sampling
(B) two stage sampling
(C) cluster sampling
(D) quota sampling

46. The arrival rate and service rates of a M/M/1 queueing system are 3 and 4, then the probability of system emptiness is
(A) 0.25 (B) 0.75
(C) 0.3 (D) 0.5

47. A simplex is a
(A) half plane
(B) convex polyhedran
(C) hull
(D) envelop

48. If A = $[a_{ij}]$ is the payoff matrix then saddle point exist when
(A) $\text{Min}_j \text{ Max}_i\, a_{ij} = \text{Max. Min } a_{ij}$
(B) $\text{Min}_j \text{ Max}_i\, a_{ij} < \text{Max. Min } a_{ij}$
(C) $\text{Min}_j \text{ Max}_i\, a_{ij} > \text{Max. Min } a_{ij}$
(D) All the above three are correct

49. If f(x) is the p.d.f of a life time distribution then the hazard rate of distribution is
(A) f(x)/F(x)
(B) f(x)/(1 – F(x))
(C) F(x)/f(x)
(D) log f(x)/(1 – F(x))

50. The variance covariance matrix of a random vector $\bar{x}$ is $\Sigma = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & 2 & 8 \end{bmatrix}$, then the first principal component explains ______ percentage of variations on the dependent variable.
(A) 90 (B) 45
(C) 50 (D) 60

Space for Rough Work