

SUBJECT CODE	SUBJECT	PAPER										
A-15-03	MATHEMATICAL SCIENCE	III										
HALL TICKET NUMBER												
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DURATION	MAXIMUM MARKS	NUMBER OF PAGES										
2 HOUR 30 MINUTES	150	16										
NUMBER OF QUESTIONS		75										

This is to certify that, the entries made in the above portion are correctly written and verified.

Candidates Signature

Name and Signature of Invigilator

Instructions for the Candidates

- Write your Hall Ticket Number in the space provided on the top of this page.
- This paper consists of seventy five multiple-choice type of questions.
- At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to **open the booklet and compulsorily examine it as below** :
 - To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
 - Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
 - After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
- Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
Example: (A) (B) (C) (D)
 where (C) is the correct response.
- Your responses to the items are to be indicated in the **OMR Answer Sheet given to you**. If you mark at any place other than in the circle in the Answer Sheet, it will not be evaluated.
- Read instructions given inside carefully.
- Rough Work is to be done in the end of this booklet.
- If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
- The candidate must handover the OMR Answer Sheet to the invigilators at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. The candidate is allowed to take away the carbon copy of OMR Sheet and used Question paper booklet at the end of the examination.
- Use only Blue/Black Ball point pen.**
- Use of any calculator or log table etc., is prohibited.**
- There is no negative marks for incorrect answers.**

అభ్యర్థులకు సూచనలు

- ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్ టికెట్ నంబరు రాయండి.
- ఈ ప్రశ్న పత్రము డెబ్బైఐదు బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
- పరీక్ష ప్రారంభమున ఈ ప్రశ్నపత్రము మీకు ఇవ్వబడుతుంది. మొదటి ఐదు నిమిషములలో ఈ ప్రశ్నపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా **సరిచూసుకోండి.**
 - ఈ ప్రశ్న పత్రమును చూడడానికి కవర్ పేజీ అంచున ఉన్న కాగితపు సీలును చించండి. స్టిక్కర్ సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నపత్రమును మీరు అంగీకరించవద్దు.
 - కవరు పేజీ పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్నపత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏదైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇప్పివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.
 - పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నపత్రము పైనిర్దిష్టస్థలంలో రాయవలెను.
- ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయ ప్రతిస్పందనలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతిప్రశ్నకు సరైన ప్రతిస్పందనను ఎన్నుకొని కింద తెలిపిన విధంగా OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన ప్రతిస్పందనను సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్ తో కింద తెలిపిన విధంగా పూరించాలి.
ఉదాహరణ : (A) (B) (C) (D)
 (C) సరైన ప్రతిస్పందన అయితే
- ప్రశ్నలకు ప్రతిస్పందనలను ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమూహన పత్రంపై వేరొక చోట గుర్తిస్తే మీ ప్రతిస్పందన మూల్యాంకనం చేయబడదు.
- ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
- చిత్తుననివి ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీస్థలములో చేయాలి.
- OMR పత్రము పై నిర్ణీత స్థలంలో సూచించవలసిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అసర్దుతకు మీరే బాధ్యులువుతారు.
- పరీక్ష పూర్తయిన తర్వాత మీ OMR పత్రాన్ని తప్పనిసరిగా పరీక్ష పర్యవేక్షకుడికి ఇవ్వాలి. వాటిని పరీక్ష గది బయటకు తీసుకువెళ్లకూడదు. పరీక్ష పూర్తయిన తరువాత అభ్యర్థులు ప్రశ్న పత్రాన్ని, OMR పత్రం యొక్క కార్బన్ కాపీని తీసుకువెళ్లనచ్చు.
- నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.
- లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్ష గదిలో ఉపయోగించడం నిషేధం.
- తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.



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MATHEMATICAL SCIENCES

Paper – III

1. The series $1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \dots$
 ∞ is

- (A) convergent
- (B) divergent
- (C) oscillatory
- (D) conditionally convergent

2. If f is defined by $f(x) = x^2 \sin \frac{1}{x}$, if $x \neq 0$
 $= 0$, if $x = 0$

then

- (A) f is differentiable at $x = 0$ but $\lim_{x \rightarrow 0} f'(x) \neq f'(0)$
- (B) f is differentiable at $x = 0$ and $\lim_{x \rightarrow 0} f'(x) = f'(0)$
- (C) f' is continuous at $x = 0$ and $f'(0) = 0$
- (D) f' is not continuous at $x = 0$ and $f'(0) = 1$

3. Which one of the following statements is NOT true ?

- (A) Every continuous function is integrable
- (B) If $|f|$ is integrable on $[a, b]$, then f is integrable on $[a, b]$
- (C) If f is continuous in $[a, b]$, then f is bounded in $[a, b]$
- (D) If f is differentiable at a point, then it is necessarily continuous at that point

4. If $[x]$ denotes the greatest integer less than or equal to x , then $\int_0^3 [x] dx =$

- (A) 3
- (B) $\frac{9}{2}$
- (C) 2
- (D) 1

5. Let f be defined by

$$f(x) = \frac{1}{2^n}, \text{ for } \frac{1}{2^{n+1}} < x < \frac{1}{2^n}; \dots, n=0,1,2, \dots$$

and $f(0) = 0$. Then consider the following statements

- I. f is integrable on $[0, 1]$
 - II. f has finite number of discontinuities in $[0, 1]$
 - III. f is monotonically increasing and bounded on $[0, 1]$
 - IV. f is continuous on $[0, 1]$
- (A) I, II and III are true
 - (B) I and II are true
 - (C) III and IV are true
 - (D) I and III are true



6. Assertion (A) : If $f(x) = \frac{1}{(x-1)^2}$, then

$$\int_0^2 f(x) dx = \left[\frac{-1}{x-1} \right]_0^2 = -2$$

is false

Reason (R) : $f'(x)$ is not bounded in $[0, 2]$

Then, which one of the following is true ?

- (A) (A) is true but (R) is false
 (B) Both (A) and (R) are false
 (C) Both (A) and (R) are true but (R) is not a correct explanation of (A)
 (D) Both (A) and (R) are true and (R) is a correct explanation for (A)
7. Consider the sequences (List I) and the properties of sequences (List II).

List I

I. $\{1 + (-1)^n\}$

II. $\{n^2\}$

III. $\{n(-1)^n\}$

IV. $\left\{1 + \frac{1}{n}\right\}$

List II

1. Oscillates infinitely

2. Converges to the limit 1

3. Diverges to $+\infty$

4. Oscillates finitely

Which one of the following options is a correct match ?

	I	II	III	IV
(A)	4	3	1	2
(B)	1	2	3	4
(C)	1	4	3	2
(D)	4	3	2	1

8. The eigen values of the matrix $\begin{bmatrix} 3 & 12 \\ 6 & 2 \end{bmatrix}$ are

- (A) $\{3, 2\}$ (B) $\{1, 1\}$
 (C) $\{11, -6\}$ (D) $\{6, -11\}$

9. The inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 5 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ is

- (A) $\frac{1}{40}(A^2 - 11A - 38I)$
 (B) $\frac{1}{40}(A^2 - 11A + 38I)$
 (C) $\frac{1}{40}(A^2 + 11A - 38I)$
 (D) $\frac{1}{40}(A^2 + 11A + 38I)$

10. If $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$, then the number of

linearly independent eigen vectors of A corresponding to the eigen value 3 is

- (A) 0 (B) 1
 (C) 2 (D) infinite

11. The rank of the matrix

$$A = \begin{bmatrix} 2 & -1 & 3 & -1 \\ -1 & 3 & 2 & -1 \\ 0 & 5 & 7 & -3 \\ 3 & 1 & -4 & 1 \end{bmatrix} \text{ is}$$

- (A) 1 (B) 2
 (C) 3 (D) 4



12. Which one of the following is the orthonormal basis for \mathbb{R}^3 (\mathbb{R}) ?

(A) $\left\{ \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right) \right\}$

(B) $\left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right), \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right) \right\}$

(C) $\left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right) \right\}$

(D) $\left\{ \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0 \right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right) \right\}$

13. If $T : V \rightarrow W$ is a linear transformation of the vector space V onto the vector space W and if $\dim. W = 4$ and $\dim. \text{Ker } T = 2$, then $\dim. V =$

- (A) 6 (B) 2
(C) 8 (D) 12

14. Let V be an n dimensional vector space over the field \mathbb{R} and $B = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a basis for V (\mathbb{R}). Then B is called an orthonormal basis for V if

- (A) $(\alpha_i, \alpha_j) = 0$ for $i \neq j, 1 \leq i, j \leq n$
(B) $(\alpha_i, \alpha_i) = 1$ for $1 \leq i \leq n$
(C) Both the options (A) and (B) hold
(D) Either option (A) or option (B) hold

15. $\int_C \frac{1}{z^2 + 4} dz, C : |z - i| = 2$ described in the positive sense, is equal to

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

16. For the function $f(z) = (z+1) \cdot \sin \frac{1}{(z-2)}$, the point $z = 2$ is

- (A) an essential singularity
(B) a pole of order one
(C) a removable singularity
(D) an analytic point

17. The sum of the residues at the poles of the function $f(z) = \frac{z^2 + 1}{z^2 - 2z}$ is

- (A) 2
(B) 3
(C) 4
(D) 6

18. The Harmonic conjugate of the function $u(x, y) = 2x - 2xy$ is

- (A) $x^2 + y^2 - 4y$
(B) $x^2 - 2xy + y^2$
(C) $2x^2 + y^2 - 3x$
(D) $x^2 - y^2 + 2y$



19. The Laurent series expansion of

$$f(z) = \frac{z^2 - 1}{z^2 + 5z + 6} \text{ about } z = 0 \text{ in the region}$$

$$2 < |z| < 3 \text{ is}$$

(A) $1 + \frac{2}{z} \left(1 + \frac{2}{z} + \frac{2^2}{z^2} + \dots \right) - \frac{5}{3} \left(1 - \frac{z}{3} + \frac{z^2}{3^2} - \dots \right)$

(B) $1 - \frac{3}{z} \left(1 - \frac{2}{z} + \frac{2^2}{z^2} - \dots \right) + \frac{5}{3} \left(1 + \frac{z}{3} - \frac{z^2}{3^2} + \dots \right)$

(C) $1 + \frac{3}{z} \left(1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \dots \right) - \frac{8}{3} \left(1 - \frac{z}{3} + \frac{z^2}{3^2} - \dots \right)$

(D) $1 + \frac{3}{z} \left(1 + \frac{2}{z} - \frac{2^2}{z^2} + \frac{2^3}{z^3} - \dots \right) - \frac{8}{3} \left(1 - \frac{z}{3} - \frac{z^2}{3^2} - \dots \right)$

20. An example of an integral domain which is not a field is

(A) $(\mathbb{Q}, +, \cdot)$

(B) $(\mathbb{Z}, +, \cdot)$

(C) $(\mathbb{Z}_6, +_6, \cdot_6)$

(D) $(\mathbb{Z}_7, +_7, \cdot_7)$

21. If ϕ is the Euler function, then $\phi(\phi(1001)) =$

(A) 720

(B) 360

(C) 224

(D) 192

22. If 6 distinct objects are in a row, then the number of possible derangements of these objects is

(A) 760

(B) 360

(C) 265

(D) 125

23. Which one of the following polynomials is irreducible over the field $(\mathbb{Z}_7, +_7, \cdot_7)$?

(A) $x^2 + 3x - 2$

(B) $x^2 + 3$

(C) $x^2 + 4x + 2$

(D) $x^2 + 5$

24. If \mathbb{Q} is the field of rational numbers, then the order of the Galois group of the polynomial $x^4 + 1 \in \mathbb{Q}[x]$ is

(A) 2

(B) 4

(C) 6

(D) 8

25. Let (X, τ) be a topological space and A, B be subsets of X . Then which one of the following statements is true ?

(A) $\overline{A \cap B} = \overline{A} \cap \overline{B}$

(B) $\overline{A - B} = \overline{A} - \overline{B}$

(C) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(D) $\overline{A'} = (\overline{A})'$

(Here \overline{A} is the closure of A and A' is the complement of A)



26. Let X be an infinite set and $\tau = \{U \subseteq X \mid U = \emptyset \text{ or } X - U \text{ is finite}\}$. Then the topological (X, τ) is a

- (A) Regular space
- (B) Normal space
- (C) Hausdorff space
- (D) T_1 space

27. A compact Hausdorff topological space is

- (A) T_3 space but not normal
- (B) normal
- (C) discrete
- (D) completely normal

28. The number of solutions of the equation $5x - 6 = 0$ in the ring $(\mathbb{Z}_{10}, +_{10}, \cdot_{10})$ of integers modulo 10 is

- (A) 0
- (B) 1
- (C) 3
- (D) 5

29. If $n(A) = 7$, then the number of bijections from A to A such that $f(x) = x$ holds exactly for 3 elements of A is

- (A) 35
- (B) 315
- (C) 1520
- (D) 2520

30. The particular integral of the equation

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x \text{ is}$$

- (A) $\sin x + x \log |\cos x|$
- (B) $-x \cos x + \sin x \log |\sin x|$
- (C) $x \sin x - \cos x \log |\sin x|$
- (D) $x \sin x + \cos x \log |\sec x|$

31. The Wronskian of the two functions

$$\phi_1(x) = x^2 \text{ and } \phi_2(x) = x|x| \text{ on } \mathbb{R} \text{ is}$$

- (A) -1
- (B) 0
- (C) 1
- (D) 2

32. The solution $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ for the system of

$$\begin{aligned} \text{equations } y_1' &= 4y_1 - 3y_2 \text{ is} \\ y_2' &= 2y_1 - y_2 \end{aligned}$$

- (A) $c_1 e^x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{2x} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- (B) $c_1 e^x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-2x} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- (C) $c_1 e^x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2x} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
- (D) $c_1 e^x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2x} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$



33. Let f be defined on the rectangle $R: |x| \leq 1, |y| \leq 1$ by

$$f(x, y) = 0, \text{ if } x = 0, |y| \leq 1$$

$$= 2x, \text{ if } 0 < |x| \leq 1, -1 \leq y < 0$$

$$= 2x - \frac{4y}{x}, \text{ if } 0 < |x| \leq 1, 0 \leq y \leq x^2$$

$$= -2x, \text{ if } 0 < |x| < 1, x^2 \leq y \leq 1$$

Then which one of the following is NOT true ?

- (A) f is continuous on \mathbb{R}
- (B) $|f(x, y)| \leq 2$ on \mathbb{R}
- (C) f satisfies the Lipschitz's condition on \mathbb{R}
- (D) $|f(x, y)| \leq 1$ on \mathbb{R}

34. The general solution of the partial differential equation $(x^2 - y^2 - yz)p + (x^2 - y^2 - 2x)q = z(x - y)$ with the usual notation, is

- (A) $f\left(x - y - z, \frac{x^2 - y^2}{z^2}\right) = 0$
- (B) $f\left(x - y + z, \frac{x^2 + y^2}{z^2}\right) = 0$
- (C) $f\left(x - y + z, \frac{x^2 - y^2}{z}\right) = 0$
- (D) $f\left(x + y - z, \frac{x^2 + y^2}{2z^2}\right) = 0$

35. The particular integral of the equation $(D^2 - D^1)z = 2y - x^2$, with the usual notation, is

- (A) xy
- (B) x^2y
- (C) xy^2
- (D) $x^2y - y^2$

36. The general solution of the equation $(y + zx)p - (x + yz)q = x^2 - y^2$, with the usual notation, is

- (A) $F(x^2 - y^2 - z^2, x + yz) = 0$
- (B) $F(x^2 + y^2 + z^2, x + z) = 0$
- (C) $F(x^2 + y^2 - z^2, xy + z) = 0$
- (D) $F(x^2 - y^2 - z^2, xy - z) = 0$

37. If $h = 1$ and Δ stands for the forward difference operator, then $\Delta^3[(1 - x)(1 - 2x)(1 - 3x)] =$

- (A) -36
- (B) -24
- (C) 12
- (D) 18

38. The estimate of error in computing $\sin 52^\circ$ from the following tabular data, using Newton-Gregory forward interpolation formula is

x	:	45°	50°	55°	60°
$y = \sin x$:	0.7071	0.7660	0.8192	0.8660

- (A) $(0.29) \times 10^{-4}$
- (B) $(0.39) \times 10^{-4}$
- (C) $(0.48) \times 10^{-4}$
- (D) $(0.54) \times 10^{-4}$



39. Given that $y' - y^2 = 0$, $y(0) = 1$, then the value of $y(0.2)$, evaluated by Euler's forward method with $h = 0.1$, correct to 3 decimal places, is

- (A) 1.143 (B) 1.176
(C) 1.221 (D) 1.265

40. The integral that should be minimized to find the curve joining the points (x_1, y_1) and (x_2, y_2) in the plane, that yields a surface of revolution with minimum area when the curve is rotated about the X-axis, is

- (A) $\int_{x_1}^{x_2} 2\pi y \sqrt{1+y'^2} dx$
(B) $\int_{x_1}^{x_2} \frac{2\pi y}{\sqrt{1+y'^2}} dx$
(C) $\int_{x_1}^{x_2} 2\pi x \sqrt{1+y'^2} dx$
(D) $\int_{x_1}^{x_2} 2\pi xy \sqrt{1+y'^2} dx$

41. A solution of the integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+t^2} y(t) dt \text{ is}$$

- (A) $y(x) = \frac{1}{\sqrt{1+x^2}}$
(B) $y(x) = \frac{x}{(1+x^2)}$
(C) $y(x) = \frac{1}{(1+x^2)^{3/2}}$
(D) $y(x) = \frac{1}{(1-x^2)^2}$

42. The number of degrees of freedom, in the case of a hoop rolling down an inclined plane without slipping, is

- (A) 1
(B) 2
(C) 3
(D) 4

43. If a force $\vec{F} = \vec{F}_x \vec{i} + \vec{F}_y \vec{j}$, where the components of force \vec{F}_x and \vec{F}_y are functions of x and y , is conservative then

- (A) $\frac{\partial F_y}{\partial y} - \frac{\partial F_x}{\partial x} = 0$
(B) $\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0$
(C) $\frac{\partial F_y}{\partial y} - \frac{\partial F_x}{\partial x} = 0$
(D) $\frac{\partial F_y}{\partial x} + \frac{\partial F_x}{\partial x} = 0$

44. A particle of mass m moves along the curve given by $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ with $0 \leq \theta \leq 2\pi$. Then the kinetic energy of that particle is

- (A) $ma^2\dot{\theta}^2(1 - \cos\theta)$
(B) $\frac{1}{2} ma^2(1 - \cos\theta)$
(C) $ma^2(1 - \cos 2\theta)$
(D) $\frac{1}{2} ma^2(1 - \cos\theta) \dot{\theta}^2$



45. The eigen values of the integral equation

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)y(t) dt \text{ are}$$

- (A) $\pm \frac{1}{2\pi}$
- (B) $\pm \frac{3}{2\pi}$
- (C) $\pm \frac{2\pi}{3}$
- (D) $\pm \frac{1}{\pi}$

46. Average strength of eleven members is 11. Average strength of first six members is 10.5. Average strength of last six members is 11.5. The value of the sixth member is

- (A) 10.5
- (B) 11.5
- (C) 11.0
- (D) 10.0

47. Let A and B be two events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$. Then $P(A \cap B)$ is

- (A) 0.9
- (B) 0.5
- (C) 0.1
- (D) 0.3

48. Which of the following is NOT a characteristic function ?

- (A) $e^{-|t|}$
- (B) e^{-t^3}
- (C) e^{-t^2}
- (D) $\frac{1}{1+t^2}$

49. Identify the correct statement

- (A) Almost sure convergence implies convergence in probability
- (B) Convergence in distribution always implies convergence in probability
- (C) Weak law of large numbers implies strong law of large numbers
- (D) Central limit theorem always implies r^{th} mean convergence

50. With usual notations $p_{ij}^{(n)} > 0$ when $n = k, 2k, 3k, \dots$ then k is called

- (A) Sampling Interval
- (B) Period
- (C) Recurrent
- (D) Transient

51. If $\lim_{n \rightarrow \infty} p_{ii}^{(n)} = p$, then the value of p lies in

- (A) $[0, 1]$
- (B) $(0, 1)$
- (C) $[0, \infty)$
- (D) $[-1, +1]$



52. The Markov chain representing number of calls coming to a cell phone per day has a
- (A) finite sample space
 - (B) countable sample space
 - (C) both (A) and (B)
 - (D) infinite sample space
53. Which of the following is a leptokurtic distribution?
- (A) normal
 - (B) student -t
 - (C) exponential
 - (D) uniform
54. The mean of a binomial variate is 4 and its variance is 3. What is its mode?
- (A) 3
 - (B) 5
 - (C) 4
 - (D) 6
55. Let X_1, X_2, \dots, X_n be iid r.v.s following Cauchy distribution with location 0 and scale 1. Then the distribution of $\bar{X} = (X_1 + X_2 + \dots + X_n)/n$ is
- (A) normal with mean 0 and variance 1
 - (B) asymptotically standard normal as $n \rightarrow \infty$
 - (C) exactly Cauchy with location 0 and scale 1
 - (D) not well defined
56. When testing null hypothesis $H_0: \mu = \mu_0$ in case of normal population with known variance against two sided alternative, the likelihood ratio tests are same as the corresponding
- (A) MP test
 - (B) UMPU test
 - (C) UMP test
 - (D) Non-parametric test
57. If (X_1, X_2, \dots, X_n) is a random sample from $U(0, \theta)$ then the maximum likelihood estimator of θ is
- (A) sample mean
 - (B) sample median
 - (C) sample minimum
 - (D) sample maximum
58. An unbiased estimator need not be
- (A) consistent
 - (B) unique
 - (C) both (A) and (B)
 - (D) neither (A) nor (B)
59. To compare the efficiency of two statistical tests we use the following function
- (A) Likelihood function
 - (B) OC function
 - (C) Probability function
 - (D) Density function



60. Which of the following is used as a test for goodness of fit ?
- (A) t-test
 - (B) F-test
 - (C) sign test
 - (D) Kolmogorov – Smirnov test
61. In ANOVA two way classification with 6 treatments and 5 replicates, the error degrees of freedom is
- (A) 18
 - (B) 19
 - (C) 20
 - (D) 21
62. The range of regression coefficients is
- (A) $[-1, +1]$
 - (B) $(-\infty, +\infty)$
 - (C) $(-1, +1)$
 - (D) $[-\infty, +\infty]$
63. In ANOVA we use the following theorem
- (A) Baye's theorem
 - (B) Factorization theorem
 - (C) Addition theorem
 - (D) Cocrran's theorem
64. Logistic regression model can be used when the data is _____ data.
- (A) nominal
 - (B) proportions
 - (C) logarithmic
 - (D) interval
65. The following technique is used for clustering the observations
- (A) Dendogram
 - (B) Stemleaf diagram
 - (C) Histogram
 - (D) Pie chart
66. Let $X = (X_1, X_2, X_3, \dots, X_p)$ be multivariate normal random variable. Then which of the following is NOT true ?
- (A) All marginal variables are univariate normal distribution
 - (B) All linear combinations of marginal variables have univariate normal distribution
 - (C) If any two marginal variables are uncorrelated then they are not independent
 - (D) The sub vector (X_1, X_2, X_3) has a trivariate normal distribution
67. Similarity between pairs of variables is measured by the
- (A) correlation coefficient
 - (B) product moment correlation coefficient
 - (C) rank correlation coefficient
 - (D) coefficient of variation



68. In the usual notation, suppose $M = X'X$, where $X(m \times p)$ follows $N_p(0, \Sigma)$. Then the distribution of M is
- (A) $W_p(m, \Sigma)$
(B) $W_p(m - 1, \Sigma)$
(C) $W_p(m, I)$
(D) $W_p(p, \Sigma)$
69. The sample size required when the population is perfectly homogeneous is
- (A) 1
(B) 100
(C) N
(D) ∞
70. If in a population the observations are heterogeneous between the groups and homogeneous within the group, then the following method of sample design is used.
- (A) simple random sampling
(B) cluster sampling
(C) stratified random sampling
(D) systematic sampling
71. In a LSD the standard error of the difference between two treatment means, none of which corresponds the missing value is given by
- (A) $S_E \sqrt{\frac{m}{2}}$
(B) $S_E \sqrt{\frac{2}{m}}$
(C) $S_E^2 \sqrt{\frac{m}{2}}$
(D) $S_E^2 \sqrt{\frac{2}{m}}$
72. In the usual notation the sum of squares due to any factorial effect is
- (A) $\frac{[]^2}{\sum \gamma_i c_i^2}$ (B) $\frac{[]^2}{\sum \gamma_i^2 c_i}$
(C) $\frac{[]^2}{\sum \gamma_i^2}$ (D) $\frac{[]^2}{\sum \gamma_i c_i}$
73. The failure time distribution which is having constant hazard rate is
- (A) normal
(B) exponential
(C) uniform
(D) log normal
74. The following method is used for obtaining the system size distribution of a $M/G/1$ queuing model
- (A) embedded Markov chain technique
(B) cluster method
(C) branch and bound method
(D) simplex method
75. For the following LPP
- Maximize: $z = x_1 + 2x_2$
- Subject to $x_1 + x_2 \geq 1$; $x_1 + 2x_2 \leq 10$
- $x_2 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$
- (A) optimum solution is unique
(B) optimum solution exist but not unique
(C) optimum solution is unbounded
(D) it can be solved using calculus with out differentiation



Space for Rough Work



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